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Electromagnetic Scattering from Generalized Impedance Cylinder by Moment Method

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Abstract

Electromagnetic scattering from generalized inhomogeneous impedance cylinder of arbitrary shape is presented for both TM and TE polarized incident field. The scattered TE and TM fields are expressed as single layer potentials. Using the boundary condition and jump relations of single layer potential on the boundary, boundary integral equation is obtained and solved via moment method. The obtained results are compared with those obtained by analytical method for inhomogeneous anisotropic impedance cylinder and good agreements are observed.

Key words: electromagnetic scattering, integral equations, impedance boundary sonditions.

1. Introduction

The boundary conditions which electric and magnetic fields have to satisfy on the surfaces of object play an important role in scattering problems. One of the this conditions is called as the impedance boundary condition (IBC) which gives a relation between tangential electric and magnetic field vectors on a given surface in terms of coefficient called surface impedance. Leontovich [1] and Wait [2] used this type boundary condition firstly. The simplest form of the IBC is SIBC which is use to model coatings and lossy dielectrics and mostly occur in our environment [3]. Generally, the surface impedance appearing in IBC is assumed to be independent of the location and associated with a constant coefficient [3, 4]. On the other hand, when a more accurate SIBC is considered, the surface impedance may be a function of location, and may be anisotropic to model anisotropic medium and corrugated surfaces. For example, when the inhomogeneous earth surface composed of different parts such as rocky soil, sand, forest, see etc. is modeled by an IBC, the surface impedance becomes a function of location. Therefore, one has to use more general IBC in order to investigate the scattering from complicated material. Scattering from canonical structures whose surfaces satisfy inhomogeneous isotropic SIBC have been proposed in [5,6,7], scattering from inhomogeneous isotropic impedance cylinder of arbitrary shape has been investigated by Nyström method in [8] for nonzero surface impedance. Scattering from anisotropic inhomogeneous impedance circular cylinder has been presented in [9] by series expansion method. Scattering from anisotropic inhomogeneous impedance cylinder of arbitrary shape is solved by physical optics (PO) method in [10]

The main objective of this study is to describe method for the solution of the direct scattering problems with objects having arbitrary shape and anisotropic inhomogeneous impedance boundary

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conditions for both TE and TM plane wave illuminations. Proposed method is based on an integral representation of the scattered TM and TE fields through single layer potential that leads to a boundary integral equation through the jump relations of single layer potentials. This integral equation is well-posed and can be solved numerically through a method of moments.

In Section 2, the scattering problem is formulated and solved. In Section 3, some examples are given. We also compare the results with those obtained by analytical technique [9] available for anisotropic inhomogeneous impedance cylinder. Both results match accurately. Finally, conclusions and concluding remarks are given in Section 4. A time factor $\exp\{-i\omega t\}$ is assumed and omitted throughout the paper.

2. Formulation and Solution of the Problem

The geometry of the considered scattering problem and parameters employed in the formulation are shown in Fig.1. The object defined by its boundary ∂D and inhomogeneous anisotropic surface impedance $\overline{\overline{Z}}(\vec{r}),\ \vec{r}\in\partial D$. The exterior environment is taken to be medium with permittivity ε, permeability μ and lossless.

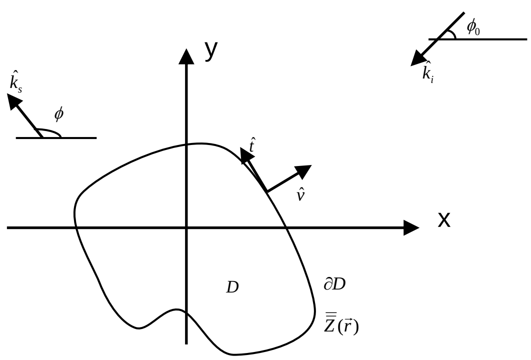

Figure 1. The geometry of the problem

Cylinder is illuminated by monochromatic plane wave whose electric field is along *z* axis that corresponds to TM illumination of the form as,

$$\begin{aligned}&\vec{E}^i(\vec{r})=(0,0,E_z^i(\vec{r}))\\&E_z^i(\vec{r})=e^{ik\hat{k}_i.\vec{r}}\end{aligned}\tag{1}$$

or whose magnetic field is along the *z* axis that corresponds to TE illumination of the form as,

$$\begin{aligned}&\vec{H}^i(\vec{r})=(0,0,H_z^i(\vec{r}))\\&H_z^i(\vec{r})=\frac{1}{Z_0}e^{ik\hat{k}_i.\vec{r}}\end{aligned}\tag{2}$$

where $Z_0 = \sqrt{\mu / \varepsilon}$ is characteristic impedance of exterior medium and $\hat{k}_i = -\cos\phi_0 \hat{u}_x - \sin\phi_0 \hat{u}_y$ is the propagation direction of incident field with incidence angle ϕ_0 and, $k = \omega\sqrt{\varepsilon\mu}$ is the wave number of exterior region. Due to the homogeneity of the problem with respect the z-axis, partial derivative with respect to z is zero. Since boundary condition is anisotropic, total field contains both TM ($E_z \neq 0, H_z = 0$) and TE ($E_z = 0, H_z \neq 0$) fields.

The TM and TE fields satisfy the reduced Helmholtz equation as

$$\Delta E_z + k^2 E_z = 0 \tag{3}$$

$$\Delta H_z + k^2 H_z = 0$$

and the inhomogeneous anisotropic IBC [9,10],

$$\hat{v}(\vec{r}) \times (\hat{v}(\vec{r}) \times \vec{E}) = -\overline{\overline{Z}}(\vec{r}).(\hat{v}(\vec{r}) \times \vec{H}(\vec{r})), \quad \vec{r} \in \partial D \tag{4}$$

and radiation conditions as

$$\lim_{r\to\infty} \sqrt{\rho}\left(\frac{\partial E_z^s}{\partial \rho} - ikE_z^s\right) = 0, \quad \rho = |\vec{r}| \tag{5}$$

$$\lim_{r\to\infty} \sqrt{\rho}\left(\frac{\partial H_z^s}{\partial \rho} - ikH_z^s\right) = 0, \quad \rho = |\vec{r}| \tag{6}$$

where $\hat{v}$ is unit normal vector on ∂D, and $\overline{\overline{Z}}(r)$ is inhomogeneous impedance dyadic expressed as,

$$\overline{\overline{Z}}(\vec{r}) = Z_{vv}(\vec{r})\hat{u}_z\hat{u}_z + Z_{zt}(\vec{r})\hat{u}_z\hat{t}(\vec{r}) + Z_{tt}(\vec{r})\hat{t}(\vec{r})\hat{t}(\vec{r}) + Z_{tz}(\vec{r})\hat{t}(\vec{r})\hat{u}_z, \quad \vec{r} \in \partial D \tag{7}$$

where $\hat{t}(\vec{r}) = \hat{u}_z \times \hat{v}(\vec{r})$ is tangential unit vector on ∂D as depicted fig.1. As seen from (7), this is most general IBC and all kind of boundary condition discussed previously in literature can be expressed by appropriate choice of impedance functions. For example, if $\overline{\overline{Z}}(\vec{r}) = 0$, boundary condition described in (7) reduces to perfect electric conductor (PEC) condition. If $Z_{zt} = Z_{tz} = 0 \text{ and } Z_{zz}, Z_{tt} \to \infty$, (7) reduces to perfect magnetic conductor (PMC). If $Z_{zt} = Z_{tz} = 0 \text{ and } Z_{zz}(\vec{r}) = Z_{tt}(\vec{r}) \neq 0, \vec{r} \in \partial D$, (7) reduces isotropic inhomogeneous IBC which is discussed in [5,8].
Let's represent the fields on the boundary ∂D as

$$\vec{E}(\vec{r}) = E_t(\vec{r})\hat{t}(\vec{r}) + E_z(\vec{r})\hat{u}_z, \quad \vec{r} \in \partial D \tag{8}$$

$$\vec{H}(\vec{r}) = H_t(\vec{r})\hat{t}(\vec{r}) + H_z(\vec{r})\hat{u}_z, \quad \vec{r} \in \partial D \tag{9}$$

Substituting (8) and (9) into (4), one obtains boundary conditions as

$$E_t(\vec{r}) = Z_{tz}(\vec{r})H_t(\vec{r}) - Z_{tt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \tag{10}$$

$$E_z(\vec{r}) = Z_{zz}(\vec{r})H_t(\vec{r}) - Z_{zt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \tag{11}$$

By using Maxwell equations, one can obtain as,

$$H_t(\vec{r}) = \frac{i}{kZ_0}\frac{\partial E_z}{\partial v}(\vec{r}), \quad \vec{r} \in \partial D \tag{12}$$

$$E_t(\vec{r}) = \frac{-iZ_0}{k}\frac{\partial H_z}{\partial v}(\vec{r}), \quad \vec{r} \in \partial D \tag{13}$$

Substituting (12) and (13) into (8) and (9), one obtain

$$\frac{-iZ_0}{k}\frac{\partial H_z}{\partial v}(\vec{r}) = Z_{tz}(\vec{r})\frac{i}{kZ_0}\frac{\partial E_z}{\partial v}(\vec{r}) - Z_{tt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \tag{14}$$

$$E_z(\vec{r}) = Z_{zz}(\vec{r})\frac{i}{kZ_0}\frac{\partial E_z}{\partial v}(\vec{r}) - Z_{zt}(\vec{r})H_z(\vec{r}), \quad \vec{r} \in \partial D \tag{15}$$

Since E_z^s and H_z^s satisfy Helmholtz equation and radiation condition, they can be expressed by single layer potential on the closed exterior of ∂D respectively as [11,12]

$$E_z^s(\vec{r}) = (S\Phi)(\vec{r}) = \int_{\partial D} \Phi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \quad \vec{r} \in R^2 \setminus \overline{\mathrm{D}} \tag{16}$$

$$H_z^s(\vec{r}) = (S\Psi)(\vec{r}) = \int_{\partial D} \Psi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \quad \vec{r} \in R^2 \setminus \overline{\mathrm{D}} \tag{17}$$

where S is single layer integral operator Φ and Ψ are unknown densities and $G(\vec{r},\vec{r}')$ is Green's function of exterior medium given by

$$G(\vec{r},\vec{r}') = \frac{i}{4}H_0^{(1)}(k|\vec{r}-\vec{r}'|) \tag{18}$$

where $H_0^{(1)}(.)$ is the Hankel function of the first kind and of order zero. Representation (16) and (17) can be used to evaluate fields and its normal derivative on boundary ∂D by using jump relation of single layer potential [11,12] as

$$E_z^s(\vec{r}) = (S\Phi)(\vec{r}) = \int_{\partial D} \Phi(\vec{r}')G(\vec{r},\vec{r}')ds(\vec{r}'), \quad \vec{r} \in \partial D \tag{19}$$

$$H_z^s(\vec{r}) = (S\Psi)(\vec{r}) = \int_{\partial D} \Psi(\vec{r}\,')G(\vec{r},\vec{r}\,')ds(\vec{r}\,'), \quad \vec{r} \in \partial D \tag{20}$$

$$\frac{\partial E_z^s}{\partial v}(\vec{r}) = (K\Phi)(\vec{r}) - \frac{1}{2}\Phi(\vec{r}) = \int_{\partial D} \Phi(\vec{r}\,')\frac{G(\vec{r},\vec{r}\,')}{\partial v(\vec{r})}ds(\vec{r}\,') - \frac{1}{2}\Phi(\vec{r}), \quad \vec{r} \in \partial D \tag{21}$$

$$\frac{\partial H_z^s}{\partial v}(\vec{r}) = (K\Psi)(\vec{r}) - \frac{1}{2}\Psi(\vec{r}) = \int_{\partial D} \Psi(\vec{r}\,')\frac{G(\vec{r},\vec{r}\,')}{\partial v(\vec{r})}ds(\vec{r}\,') - \frac{1}{2}\Psi(\vec{r}), \quad \vec{r} \in \partial D \tag{22}$$

substituting (19),(20),(21) and (22) into (14) and (15), boundary integral equation is obtained as

$$\frac{-iZ_0}{k}((K\Psi)(\vec{r}) - \frac{1}{2}\Psi(\vec{r})) - Z_{tz}(\vec{r})\frac{i}{kZ_0}((K\Phi)(\vec{r}) - \frac{1}{2}\Phi(\vec{r})) + Z_{tt}(\vec{r})(S\Psi)(\vec{r}) = f(\vec{r}), \quad \vec{r} \in \partial D \tag{23}$$

$$(S\Phi)(\vec{r}) - Z_{zz}(\vec{r})\frac{i}{kZ_0}((K\Phi)(\vec{r}) - \frac{1}{2}\Phi(\vec{r})) + Z_{zt}(\vec{r})(S\Psi)(\vec{r}) = g(\vec{r}), \quad \vec{r} \in \partial D \tag{24}$$

where $f(\vec{r})$ and $g(\vec{r})$ are functions depends on illumination. For TM illumination case,

$$f(\vec{r}) = -(Z_{tz}(\vec{r})/Z_0)\hat{v}(\vec{r}).\hat{k}_i e^{ik\hat{k}_i.\vec{r}}, \quad g(\vec{r}) = -((Z_{zz}(\vec{r})/Z_0)\hat{v}(\vec{r}).\hat{k}_i + 1)e^{ik\hat{k}_i.\vec{r}}, \quad \vec{r} \in \partial D \tag{25}$$

For TE illumination case,

$$f(\vec{r}) = -(\hat{v}(\vec{r}).\hat{k}_i + Z_{tt}(\vec{r})/Z_0)e^{ik\hat{k}_i.\vec{r}}, \quad g(\vec{r}) = -(Z_{zt}(\vec{r})/Z_0)e^{ik\hat{k}_i.\vec{r}}, \quad \vec{r} \in \partial D \tag{26}$$

Integral equations (23) and (24) can be approximated by a matrix equation through discretization using the point-matching MoM technique [13]. If the boundary of the object is meshed into a total of N elements, we can approximate the integral equation by the matrix equation given in a compact form as

$$\begin{bmatrix} C & D \\ E & F \end{bmatrix}\begin{bmatrix} \Psi \\ \Phi \end{bmatrix} = \begin{bmatrix} G \\ H \end{bmatrix} \tag{27}$$

where C, D, E and F are N by N matrixes and G and H are N-dimensional column vectors.

Once the boundary integral equations (23) and (24) are solved the near and far fields of the TM and TE scattered waves can be calculated through (19) and (20) respectively. Let's call TM and TE fields as vertically (V) and horizontally (H) waves respectively. Polarimetric scattering width $\sigma_{ab}, a = H,V; b = H,V$ is related to ratio of a polarized scattered power to b polarized incident power and defined by

$$\sigma_{ab}(\phi) = \lim_{\rho\to\infty} 2\pi\rho \frac{|E_a(\rho,\phi)|^2}{|E_b|^2}, \quad a = H,V; b = H,V \tag{28}$$

3. Numerical Results

The proposed procedure has been applied to two illustrative examples. In all examples, wave number of exterior medium and incidence angle are chosen as $k = 1$ and $\phi_0 = 0^0$ respectively. Integral equations (23) and (24) are solved for $N = 150$. Scattering from anisotropic inhomogeneous impedance circular cylinder whose analytical solution is available in [9] is considered. Obtained results are compared with those obtained by analytical method in [9] and good agreements are observed.

Scattering from anisotropic inhomogeneous impedance circular cylinder with radius 1m is considered for TM incident case. Parameterization of boundary and anisotropic surface impedances are given respectively as $\partial D = \{r(t) = (\cos t, \sin t), t \in [0, 2\pi)\}$ and $Z_{zz}(t) = 100(1+i)\cos(t)$, $Z_{zt}(t) = 50(1+2i)\sin(2t)$ $Z_{tz}(t) = 50(2+i)\cos(2t), Z_{tt}(t) = 100(1+i\sin t)$. $t \in [0, 2\pi)$. Since the incident field is TM wave, TM and TE scattered widths are called as σ_{VV} and σ_{HV} and depicted in fig.2 and fig.3 respectively. Obtained results are compared with those obtained by analytical method in [9]. As seen from Fig.2 and Fig.3, good agreements are observed.

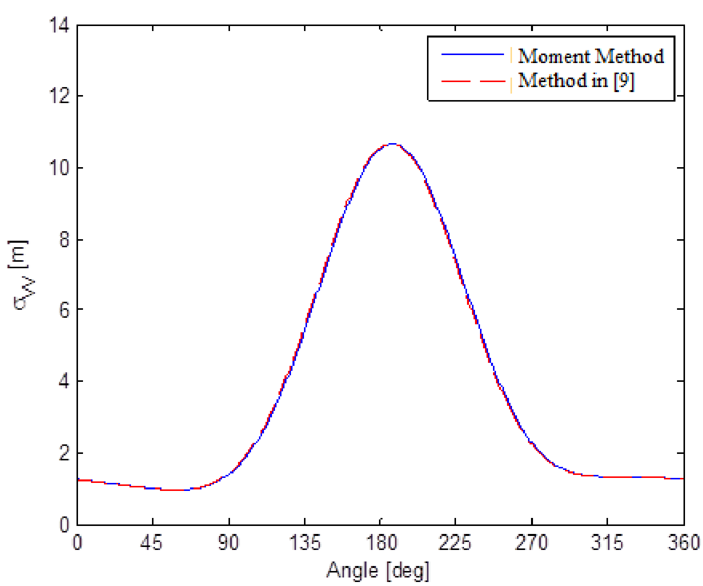

Figure 2. Scattering width σ_{VV} of the circular cylinder for case 1

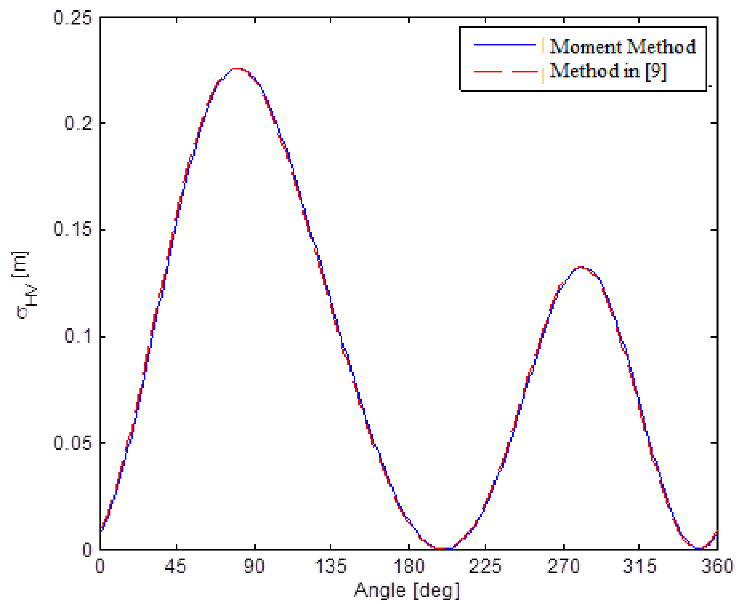

Figure 3. Scattering width σ_{HV} of the circular cylinder

4.Conclusions

According to the geometrical and physical properties of the scattering bodies the surface impedance can be a function of location and anisotropic. The problems involving anisotropic inhomogeneous IBC are of importance from both mathematical and physical points of view. In this study, electromagnetic scattering from anisotropic inhomogeneous impedance cylinder of arbitrary shape is considered by integral equations which are solved by numerical effective moment method for both TM and TE illumination. For this reason, the scattered TM and TE fields are represented by single-layer potentials firstly. Using the jump relations of single layer potentials and its normal derivatives on the boundary, boundary integral equations are obtained and solved by numerical moment method. Obtained results are compared with those obtained by analytical method for circular cylinder with anisotropic inhomogeneous IBC and good agreements are observed.

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